

ANNEX

$$F1 : \quad C = \nabla \Phi^t \nabla \Phi.$$

$$5 \quad F2 : \quad \Phi(X) = \sum_{j=0}^2 \Lambda_j(X) Q_j \Rightarrow \nabla \Phi = \sum_{j=0}^2 \alpha_j \otimes Q_j \text{ where } \alpha_j = \frac{l_j}{2A} n_j.$$

$$F3 : \quad C = \sum_{j,k} (Q_j \cdot Q_k) (\alpha_j \otimes \alpha_k).$$

$$10 \quad F4 : \quad C = \sum_j r^2 (\alpha_j \otimes \alpha_j) + \sum_{\substack{j,k \\ j \neq k}} \left(r^2 - \frac{I_p^2}{2} \right) (\alpha_j \otimes \alpha_k) \\ = r^2 \sum_{j,k} (\alpha_j \otimes \alpha_k) - \frac{1}{2} \sum_{\substack{j,k \\ j \neq k}} I_p^2 (\alpha_j \otimes \alpha_k) \\ 15 \quad = -\frac{1}{2} \sum_{\substack{j,k \\ j \neq k}} I_p^2 (\alpha_j \otimes \alpha_k)$$

$$20 \quad F5 : \quad W = \frac{\lambda}{2} (tr E)^2 + \mu tr E^2 \\ tr E = \frac{1}{2} \boxed{tr C} - 1 \\ tr E^2 = \frac{1}{4} \boxed{tr C^2} - tr E - \frac{1}{2}$$

$$25 \quad F6 : \quad tr C = -\frac{1}{2} \sum_{\substack{j,k \\ j \neq k}} I_p^2 (\alpha_j \cdot \alpha_k).$$

$$F7 :$$

$$\overrightarrow{P_p P_q} \cdot \overrightarrow{P_p P_r} = \begin{cases} l_j l_k \cos(\theta_p) = \frac{1}{2} (l_j^2 + l_k^2 - l_p^2) = K_p & \text{if } q \neq r, j = \text{comp}(p, q) \\ & \text{and } k = \text{comp}(p, r) \\ l_j^2 & \text{if } q = r, j = \text{comp}(p, q) \end{cases}$$

$$30 \quad R1 : \quad (\alpha_j \cdot \alpha_k) = \begin{cases} -\frac{l_j l_k}{4A^2} \cos(\theta_p) = -\frac{1}{4A^2} K_p & \text{if } j \neq k \text{ and } p = \text{comp}(j, k), \\ \frac{l_j^2}{4A^2} & \text{if } j = k \end{cases}$$

$$\text{F8 :} \quad \text{tr } C = \frac{1}{4A^2} \sum_p^{26} L_p^2 K_p.$$

$$\text{F9 :}$$

$$\text{tr } E = \frac{1}{8A^2} \sum_p (L_p^2 - l_p^2) K_p = \frac{1}{8A^2} [(L_0^2 - l_0^2) K_0 + (L_1^2 - l_1^2) K_1 + (L_2^2 - l_2^2) K_2]$$

$$5$$

$$\text{F10 :} \quad \text{tr } C^2 = \frac{1}{4} \sum_{\substack{j,k \\ j \neq k}} \sum_{\substack{m,n \\ m \neq n}} L_p^2 L_q^2 (\alpha_j \cdot \alpha_n) (\alpha_k \cdot \alpha_m)$$

$$\text{F11 :}$$

$$\begin{aligned} \mathbf{F}_0^T = \frac{\partial W}{\partial \mathbf{Q}_0} = & \frac{1}{32A^4} \left[(\lambda + \mu) \left(\sum_p (L_p^2 - l_p^2) K_p \right) \left(\overrightarrow{\mathbf{Q}_0 \mathbf{Q}_0} K_1 + \overrightarrow{\mathbf{Q}_1 \mathbf{Q}_0} K_2 \right) \right. \\ & + \mu \left((L_1^2 - l_1^2) l_0^2 (\overrightarrow{l_2 \mathbf{Q}_2 \mathbf{Q}_0} - \overrightarrow{\kappa_0 \mathbf{Q}_1 \mathbf{Q}_0}) + (L_2^2 - l_2^2) l_0^2 (\overrightarrow{l_1 \mathbf{Q}_1 \mathbf{Q}_0} - \overrightarrow{\kappa_0 \mathbf{Q}_2 \mathbf{Q}_0}) \right. \\ & \left. \left. - (L_0^2 - l_0^2) (l_1^2 K_1 \overrightarrow{\mathbf{Q}_1 \mathbf{Q}_0} + l_2^2 K_2 \overrightarrow{\mathbf{Q}_2 \mathbf{Q}_0}) \right) \right] \end{aligned}$$

$$10 \quad \text{F12 :}$$

$$\nabla \Phi = \sum_{j=0}^3 \alpha_j \otimes \mathbf{Q}_j \quad \alpha_j = \frac{A_j}{3V} \mathbf{n}_j$$

$$\text{F13 :}$$

$$C = \sum_{j,k} (\mathbf{Q}_j \cdot \mathbf{Q}_k) (\alpha_j \otimes \alpha_k)$$

$$15 \quad \text{F14 :}$$

$$(\mathbf{Q}_j \cdot \mathbf{Q}_k) = \begin{cases} r^2 - \frac{L_{jk}^2}{2} & \text{if } j \neq k \\ r^2 & \text{if } j = k \end{cases} \quad \Rightarrow \quad \boxed{C = -\frac{1}{2} \sum_{\substack{j,k \\ j \neq k}} L_p^2 (\alpha_j \otimes \alpha_k)}$$

$$\text{F15 :}$$

$$W = \frac{\lambda}{2} (\text{tr } E)^2 + \mu \text{tr } E^2$$

$$\text{tr } E = \frac{1}{2} \boxed{\text{tr } C} - \frac{3}{2}$$

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$$\text{tr } E^2 = \frac{1}{4} \boxed{\text{tr } C^2} - \text{tr } E - \frac{3}{4}$$

F16 :

$$\begin{aligned}
 (\alpha_j \cdot \alpha_k) &= \begin{cases} -\frac{1}{9\sqrt{2}} \mathcal{K}_{jk} & \text{if } j \neq k \\ \frac{1}{9\sqrt{2}} \mathcal{A}_j^2 & \text{if } j = k \end{cases} \\
 \text{with } \mathcal{K}_{jk} &= \frac{2 \, l_{jm}^2 (l_{jl}^2 + l_{kl}^2 - l_{jk}^2) - (l_{jl}^2 + l_{lm}^2 - l_{jm}^2)(l_{kl}^2 + l_{lm}^2 - l_{km}^2)}{16}
 \end{aligned}$$

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$$F17 : \quad \text{tr } C = \frac{1}{9\sqrt{2}} \sum_{\text{Edges } (j,k)} L_{jk}^2 \mathcal{K}_{jk}$$

15 F18 :

$$\begin{aligned}
 \text{tr } C^2 &= \frac{1}{81\sqrt{4}} \left[\frac{1}{2} \sum_{\text{edges } (j,k)} L_{jk}^4 (\mathcal{K}_{jk}^2 + \mathcal{A}_j^2 \mathcal{A}_k^2) \right. && 6 \text{ terms.} \\
 &+ \sum_{\substack{\text{pairs of edges} \\ ((j,k),(j,m))}} L_{jk}^2 L_{jm}^2 (\mathcal{K}_{jk} \mathcal{K}_{jm} - \mathcal{A}_j^2 \mathcal{K}_{km}) && 12 \text{ terms} \\
 &+ \left. \sum_{\substack{\text{pairs of edges} \\ ((j,k),(m,n))}} L_{jk}^2 L_{mn}^2 (\mathcal{K}_{jm} \mathcal{K}_{kn} + \mathcal{K}_{jm} \mathcal{K}_{kn}) \right] && 3 \text{ terms}
 \end{aligned}$$

F19 :

$$W = \frac{1}{324\sqrt{4}} \left\{ \left(\frac{\lambda + \mu}{2} \right) \left[\sum_{\text{Edges } (j,k)} (L_{jk}^2 - l_{jk}^2) \mathcal{K}_{jk} \right]^2 + \mu \bar{\Delta} \right\}$$

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F20 :

$$\begin{aligned}
 \bar{\Delta} &= \frac{1}{2} \sum_{\text{edges } (j,k)} (L_{jk}^2 - l_{jk}^2)^2 \mathcal{A}_j^2 \mathcal{A}_k^2 \\
 &- \sum_{\substack{\text{pairs of edges} \\ ((j,k),(j,m))}} (L_{jk}^2 - l_{jk}^2)(L_{jm}^2 - l_{jm}^2) \mathcal{A}_j^2 \mathcal{K}_{km} \\
 &+ \sum_{\substack{\text{pairs of edges} \\ ((j,k),(m,n))}} (L_{jk}^2 - l_{jk}^2)(L_{mn}^2 - l_{mn}^2) (\mathcal{K}_{jm} \mathcal{K}_{kn} + \mathcal{K}_{jm} \mathcal{K}_{kn} - \mathcal{K}_{jk} \mathcal{K}_{mn})
 \end{aligned}$$

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F21 :

$$\begin{aligned}
 \mathbb{F}_p = \frac{1}{162 \, V^4} & \left\{ (\lambda + \mu) \left[\sum_{\text{Edges}(j,k)} (L_{jk}^2 - l_{jk}^2) \, \kappa_{jk} \right] \left(\sum_{\text{Edges}(j,p)} \overrightarrow{Q_j Q_p} \, \kappa_{jp} \right) \right. \\
 & + \mu \left[\sum_{\text{Edges}(j,p)} (L_{jp}^2 - l_{jp}^2) \, \mathcal{A}_j^2 \left(\overrightarrow{Q_j Q_p} \, \mathcal{A}_j^2 - \overrightarrow{Q_k Q_p} \, \kappa_{jk} - \overrightarrow{Q_m Q_p} \, \kappa_{jm} \right) \right. \\
 & \left. \left. + \sum_{\substack{\text{Edges}(j,k) \\ j \neq p}} (L_{jk}^2 - l_{jk}^2) \left(\overrightarrow{Q_m Q_p} \, \kappa_{jkm} - \overrightarrow{Q_j Q_p} \, \mathcal{A}_j^2 \, \kappa_{kp} - \overrightarrow{Q_k Q_p} \, \mathcal{A}_k^2 \, \kappa_{jp} \right) \right] \right\}
 \end{aligned}$$

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$$\begin{aligned}
 \text{R2 : } \quad W &= \frac{1}{128 A^4} \left[(\lambda + \mu) \left(\sum_p (L_p^2 - l_p^2) \, \kappa_p \right)^2 \right. \\
 & \left. + \mu \left(\sum_p (L_p^2 - l_p^2)^2 \, l_p^2 - 2 \sum_{\substack{j,k \\ j \neq k}} (L_j^2 - l_j^2) (L_k^2 - l_k^2) \, l_k^2 \, \kappa_p \right) \right] \\
 p &= \text{comp}(j, k)
 \end{aligned}$$

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$$\text{E1 : } \quad m_i \frac{d^2 \mathbf{P}_i}{dt^2} = \gamma_i \frac{d \mathbf{P}_i}{dt} + \mathbf{F}_i$$

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$$\text{E2 : } \quad M \ddot{\mathbf{U}} + C \dot{\mathbf{U}} + K \mathbf{U} = \mathbf{R}$$

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$$\text{E3 : } \quad \left(\frac{m_i}{\Delta t^2} - \frac{\gamma_i}{2\Delta t} \right) P_i^{t+1} = F_i + \frac{2m_i}{\Delta t^2} P_i^t - \left(\frac{m_i}{\Delta t^2} + \frac{\gamma_i}{2\Delta t} \right) P_i^{t-1}$$

E3 :

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